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Magnetic Fields at First Order Phase Transition: A Threat to Electroweak Baryogenesis

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Abstract

The generation of the observed baryon asymmetry may have taken place during the electroweak phase transition, thus involving physics testable at LHC, a scenario dubbed electroweak baryogenesis. In this paper we point out that the magnetic field which is produced in the bubbles of a first order phase transition endangers the baryon asymmetry produced in the bubble walls. The reason being that the produced magnetic field couples to the sphaleron magnetic moment and lowers the sphaleron energy; this strengthens the sphaleron transitions inside the bubbles and triggers a more effective wash out of the baryon asymmetry. We apply this scenario to the Minimal Supersymmetric extension of the Standard Model (MSSM) where, in the absence of a magnetic field, successful electroweak baryogenesis requires the lightest CP-even Higgs and the right-handed stop masses to be lighter than about 127 GeV and 120 GeV, respectively. We show that even for moderate values of the magnetic field, the Higgs mass required to preserve the baryon asymmetry is below the present experimental bound. As a consequence electroweak baryogenesis within the MSSM should be confronted on the one hand to future measurements at the LHC on the Higgs and the right-handed stop masses, and on the other hand to more precise calculations of the magnetic field produced at the electroweak phase transition.

1 Introduction

Electroweak baryogenesis (EWBG) [1, 2] is a very elegant mechanism for generating the baryon asymmetry of the Universe (BAU). It relies on physics at the weak scale and can therefore be tested at present accelerator energies, in particular at the LHC. During the electroweak phase transition (EWPT) bubbles of the broken phase are nucleated and expand. Particles in the plasma are reflected off the bubble walls where CP is violated and CP-violating currents may be generated. If the currents efficiently diffuse into the unbroken phase they may be converted into a baryon asymmetry by the action of the baryon number violating sphaleron processes [3]. The baryon asymmetry then flows into the interior of the bubble where it is preserved provided that the sphaleron interactions are sufficiently switched off in the broken phase which defines a strong enough first order phase transition.

In this work we point out an effect that seems to have passed unnoticed so far and that may require a stronger first order phase transition, for EWBG purposes. During the first order phase transition magnetic fields are unavoidably generated [4]. Bubble collisions generate a level of turbulence and hence vorticity in the fluid. The turbulent conducting fluid develops magnetic turbulence resulting in magnetic fields on all scale sizes. The turbulence in the fluid amplifies whatever seed fields are present to finite-amplitude large-scale size magnetic fields [5]. The relevant time scale for the amplification of fields on length scale ℓ is of order $(\ell/R_{\rm b})t_{\rm pt}$, where $R_{\rm b}$ is the radius of the bubble moving with velocity $v_{\rm w}$ and $t_{\rm pt} \sim R_{\rm b}/v_{\rm w}$ is the duration of the phase transition. If the field growth is exponential fields on scales $\ell \lesssim R_{\rm b}$ can be amplified by many e-folds [6, 7]. When the magnetic turbulence becomes fully developed the kinetic energy of the turbulent flow is equipartitioned with that of the magnetic field energy implying that the magnetic fields $B(R_{\rm b})$ on the size of the bubble radius is $B^2/2 = \mathcal{O}(v_{\rm f}^2)\rho_\gamma$, where $\rho_\gamma \simeq (\pi^2 g_*/30)T^4$ is the energy density of the electroweak plasma carrying g_* relativistic degrees of freedom, T is the temperature and $v_{\rm f}$ is the fluid velocity [6, 7]. This means that a magnetic field of size

$$B \sim 0.4 \left(\frac{v_{\rm f}}{0.05}\right) T^2\,, \tag{1.1}$$

can be generated via turbulence.

One possible mechanism for generation of magnetic seed fields is by the dipole electromagnetic charge layer that develops on the surfaces of the bubbles as a consequence of baryon asymmetry [6, 7]. The rotation of the dipole charge layer thus sets up a current in the fluid. The magnetic field may be generated from these currents in the bubble walls and then be amplified to the equipartition value (1.1) by exchange of energy with the turbulent fluid. Higgs phase gradients can also act as a source for gauge fields at bubble collisions [8, 9]. Of course magnetic fields from the electroweak transition can survive only on scales on which magnetic diffusion has not had time to wash out the field correlations. This means that magnetic fields on the bubble radius scale $R_{\rm b}$ dies off on a time scale $\sim \sigma R_{\rm b}^2$, where $\sigma \sim 10\,T$ [10] is the conductivity of the plasma. Being bubble sizes $\mathcal{O}(10^{-2} - 10^{-3})$ of the Hubble radius at the electroweak phase transition magnetic fields set up at the electroweak phase transition die

away on time scales much larger than the Hubble time at the phase transition. Lacking a more precise calculation about the exact magnitude of the generated magnetic field at the electroweak phase transition we will parametrize it through the dimensionless parameter b as

$$B = b\,T^2\,, \tag{1.2}$$

while from what we have just discussed above values of $b \lesssim 0.4$ seem quite plausible. Such values are comfortably smaller than those deduced by imposing that the energy density stored in the magnetic field at the characteristic scale of production (in our scale the bubble radius) does not appreciably alter the dynamics of primordial nucleosynthesis and the structure of the CMB anisotropies [11].

Now the key point is that sphaleron configurations do possess a magnetic dipole moment [12]. In the background of a magnetic field in the bubble of the broken phase the coupling with the dipole moment lowers the height of the sphaleron barrier so that thermal fluctuations are more effective in producing topological transitions [13]. Therefore in order to preserve the baryon asymmetry within the bubble where a magnetic field is produced by the phase transition it is then necessary to require a stronger first order phase transition than in the case in which one neglects the presence of the magnetic field.

The previous comments do apply to any first order phase transition generating EWBG. However it has been shown that EWBG cannot be realized within the Standard Model (SM) framework [14–18] and it is neither feasible in its Minimal Supersymmetric extension (MSSM) for arbitrary values of its parameters [19–21]. A particular region in the space of supersymmetric mass parameters was found in the MSSM, dubbed under the name of light stop scenario (LSS) [22–44], where EWBG had the potential of being successful. In particular the condition that sphaleron interactions are inhibited in the broken phase required a sufficiently strong first order phase transition imposing absolute upper bounds on the lightest CP-even Higgs and right-handed stop masses, $m_H \lesssim 127$ GeV and $m_{\widetilde{t}_R} \lesssim 120$ GeV [44]. This is the so-called MSSM baryogenesis window.

In this paper we will consider the EWPT in the MSSM and re-analyze the EWBG constraints taking into account the magnetic field produced by the phase transition bubbles. It is then clear that the presence of a magnetic field inside the bubble combined with a non-vanishing magnetic dipole moment of the sphaleron will lower the upper bound on the lightest CP-even Higgs mass and might close the present EWBG window. However as we have previously stated in the absence of a precise enough calculation we will parametrize the magnetic field by the dimensionless parameter in Eq. (1.2) so that the results in this paper could be interpreted as upper bounds on the magnitude of the parameter b. In this way the MSSM EWBG scenario can only be disproved by future experimental results on the Higgs and right-handed stop masses and/or a more precise theoretical calculation of the produced magnetic field.

The paper is organized as follows. In Section 2 we provide a short summary of the LSS, while in Section 3 we briefly discuss the energy of the sphaleron in a magnetic field, deferring the technical details to Appendices A and B. Section 4 contains our numerical results and

Section 5 the conclusions.

2 The Light Stop Scenario and EWBG

While in the SM there exists no viable electroweak scale mechanism to explain the BAU [14–18] in the MSSM it is possible to generate the observed matter-antimatter asymmetry via EWBG [22–44]. The reason is that the LSS can overcome the two main problems precluding EWBG to work within the SM: the impossibility of a strong first order EWPT and the lack of large CP-violating sources at the EW scale. Concerning the latter the MSSM naturally provides new CP-violating interactions. If the charginos and neutralinos are light and their mass parameters have non-negligible relative phases the currents associated to them are sufficient to produce enough CP violation during the EWPT. On the other hand these phases affect observables as electric dipole moments (EDM) which are constrained by experiments. The one-loop contributions to the EDM may be efficiently suppressed if the first and second generation scalar particles have a mass equal or larger than $\mathcal{O}(10)$ TeV. Nevertheless two-loop corrections involving the charginos and the Higgs field would remain sizable unless the CP-odd Higgs mass is heavier than $\mathcal{O}(1)$ TeV. Still, even for large CP-odd Higgs mass, a contribution induced by the SM-like Higgs cannot be avoided what becomes a LSS prediction that can be tested at the forthcoming experiments [45].

Regarding the lack of a strong first order EWPT in the SM, extra bosons can strengthen the phase transition if their couplings to the Higgs are sizable and their thermal abundances are not Boltzmann suppressed. In the MSSM the scalars fulfilling these requirements are the superpartners of the top quark. In practice only the (mainly) right-handed stop may be light. In fact the heaviest (mainly) left-handed stop has to acquire a mass above a few TeV to achieve agreement with electroweak precision tests and to ensure a sufficiently heavy SM-like Higgs boson [40] compatible with the LEP bound $m_H > 114.4$ GeV [46]. On the other hand light gluinos jeopardize the improvement on the phase transition since their presence in the plasma increases substantially the thermal mass of the right-handed stops which then may become Boltzmann suppressed which implies that a gluino mass $\gtrsim$500 GeV is preferred. Finally in order to counteract the remaining thermal mass contributions to the lightest stop a negative stop square mass term is required: *i.e.* the right-handed stop is required to be lighter than the top.

In conclusion the spectrum of the LSS at the $\mathcal{O}(100)$ GeV scale appears as constituted of the SM spectrum and light charginos, neutralinos and the right-handed stop. The other fields, namely gluinos and the remaining scalars, can be decoupled because the EWBG mechanism is sensitive only to the EW scale. Therefore the generation of the BAU can be investigated in the low-energy effective theory where heavy fields are integrated out and their large radiative corrections are resummed by assuming (for simplicity) a similar mass $\widetilde{m}$ for the heavy scalars [40].

At low energy the scalar sector is described by the effective potential of the SM-like Higgs

H and the lightest stop $\widetilde{t}_R$. At finite temperature T we approximate it as

$$V(H, \widetilde{t}_R, T) = V^{(\rm tree)}(H, \widetilde{t}_R) + V^{(\rm rad)}(H, \widetilde{t}_R, T) \ , \tag{2.1}$$

where $V^{(\rm rad)}$ includes up to two-loops corrections in the top Yukawa and strong gauge couplings [1]. In fact the potential $V(H, \widetilde{t}_R, T)$ is required to analyze the phase transition. By using the bounce method one can determine the nucleation temperature T_n [47,48] below which the bubbles containing the electroweak breaking phase can form, expand and then collide to each other. This process goes on till the temperature T_f $(< T_n)$ when the Universe is completely filled of the electroweak broken phase and the transition is ended.

The successful production of the BAU can be synthetized into three main conditions. First, bubbles of electroweak broken phase must nucleate and fill the Universe. Second, the baryon asymmetry must be produced and injected into the bubbles of the broken phase. Third, inside the bubbles $SU(2)_L$ sphalerons must be out of equilibrium not to wash out the baryon asymmetry. In particular, in the limit of vanishing magnetic field $B \to 0$ three main implications follow [44]:

1. Since the square mass term of the lightest stop is negative $V(H, \widetilde{t}_R, T = 0)$ contains a color-breaking minimum along the direction $(H = 0, \widetilde{t}_R)$ besides the standard minimum at $(H = (0, v/\sqrt{2})^T, \widetilde{t}_R = 0)$ [2]. Therefore it is possible that portions of the Universe decay into the color breaking phase from where they cannot escape. To avoid such a problem the transition towards the EW breaking phase must end before any tunneling into the color breaking minimum is allowed. This requirement is roughly guaranteed by the condition $T_c \gtrsim T_{\rm col} + 1.6$ GeV, where T_c and $T_{\rm col}$ are the temperatures at which the minimum of the unbroken phase is degenerate with the electroweak and color breaking ones, respectively.

2. The amount of baryon asymmetry injected into the bubbles is enhanced nearby a resonance region occurring for degenerate gaugino and Higgsino masses [37–43]. In such a case, requiring enough BAU implies a bound on the ratio between the VEVs of the up and down Higgses of the MSSM, precisely $\tan\beta \lesssim 15$. The same bound provides suppression of the EDM within the experimental constraints.

3. The condition that sphaleron transitions are inhibited in the broken phase requires a sufficiently strong first order phase transition. Quantitatively this depends on the relation between $V(H, \widetilde{t}_R = 0, T)$ at $T \lesssim T_n$ and the $SU(2)_L$ sphaleron rate. This requirement imposes absolute upper bounds on the lightest SM-like Higgs and right-handed stop masses as $m_H \lesssim 127$ GeV and $m_{\widetilde{t}_R} \lesssim 120$ GeV [44]. These extreme values are obtained for $\widetilde{m}$ beyond the PeV scale while smaller values of $\widetilde{m}$ provide smaller

[1] To calculate $V(H, \widetilde{t}_R, T)$ we choose the low energy spectrum obtained in Ref. [40,44] to which we refer for the explicit expressions of $V^{(\rm tree)}(H, \widetilde{t}_R)$ and $V^{(\rm rad)}(H, \widetilde{t}_R, T)$.

[2] We use the convention $v \equiv v(T = 0) = 246.2\,$GeV with $v(T)$ being the Vacuum Expectation Value (VEV) of the Higgs in the EW-breaking minimum at temperature T.

upper bounds. In particular for $\widetilde{m} \lesssim 6$ TeV they lie under the experimental constraints: $m_H > 114.4$ GeV [46] and $m_{\widetilde{t}_R} \gtrsim 95$ GeV [49].

However one must be worried about the formation of the magnetic field that may substantially modify these conclusions.

As explained in the introduction the dynamics of the phase transition may enhance the seeds of the magnetic fields to sizeable values [6,7]. Its magnitude is small at bubble nucleation so that nucleation properties are not altered by it. Instead the magnetic field may become sizable during the phase transition and we consider it as a homogeneous background field B in the whole interior of the bubble. A large B affects the effective potential V [50] but the effect can be neglected because it arises beyond the approximation we use to calculate $V(H, \widetilde{t}_R, T)$ of Eq. (2.1). In the approximation we are using, where gauge boson loops play a subleading role, the gauge dependence of the effective potential [51] would affect only mildly the sphaleron energy and its effects should be comparable to other subleading effects we have not considered. Finally we assume that B does not modify significantly the diffusion processes injecting the baryon asymmetry into the interior of the bubble. For this reason we conclude that the presence of the magnetic field B threatens EWBG in the LSS through alterations of the above third implication. In fact, as we will explain in detail in the next section, the magnetic field changes the standard link between $SU(2)_L$ sphaleron rate and $V(H, \widetilde{t}_R, T)$ thus reducing the parameter space where the phase transition is strong.

3 Sphaleron in a magnetic field

For vanishing weak mixing angle, $\theta_{\rm w} = 0$, the sphaleron solution is spherically symmetric and does not develop any magnetic dipole moment. For $\theta_{\rm w} \neq 0$ the $U(1)_Y$ gauge field a_μ is excited and the spherical symmetry reduces to an axial symmetry. A magnetic dipole moment is present and can be computed at the lowest order in $\theta_{\rm w}$ using the sphaleron solutions obtained for $\theta_{\rm w} = 0$ [12]. Because of the very weak dependence on $\theta_{\rm w}$ the discrepancy with respect to computing the dipole moment with the sphaleron solutions at $\sin\theta_{\rm w} \simeq 0.48$ is less than 1% [52]. A description of the sphaleron solutions and their energy can be found in Appendix A, while the sphaleron magnetic dipole moment is discussed in Appendix B.

In the presence of a magnetic field, Eq. (1.2), inside the bubbles the sphaleron energy changes due to the interaction of the magnetic field with the sphaleron magnetic dipole moment

$$E_{\rm sph}(T, B) = E_{\rm sph}(T) + E_{\rm dipole}(T, B)\,, \tag{3.1}$$

where $E_{\rm sph}(T)$ is the sphaleron energy computed without magnetic field, but taking into account both the temperature dependence of the Higgs potential and the sphaleron solutions for $\theta_{\rm w} \neq 0$ [see Eq. (A.17)]. The dipole energy to leading order in $\theta_{\rm w}$ is computed using Eqs. (B.4)-(B.5)

$$E^{(1)}_{\rm sph}(T, B) = E_{\rm sph}(T) + E^{(1)}_{\rm dipole}(T, B) = E_{\rm sph}(T) - \mu(T)B\,. \tag{3.2}$$

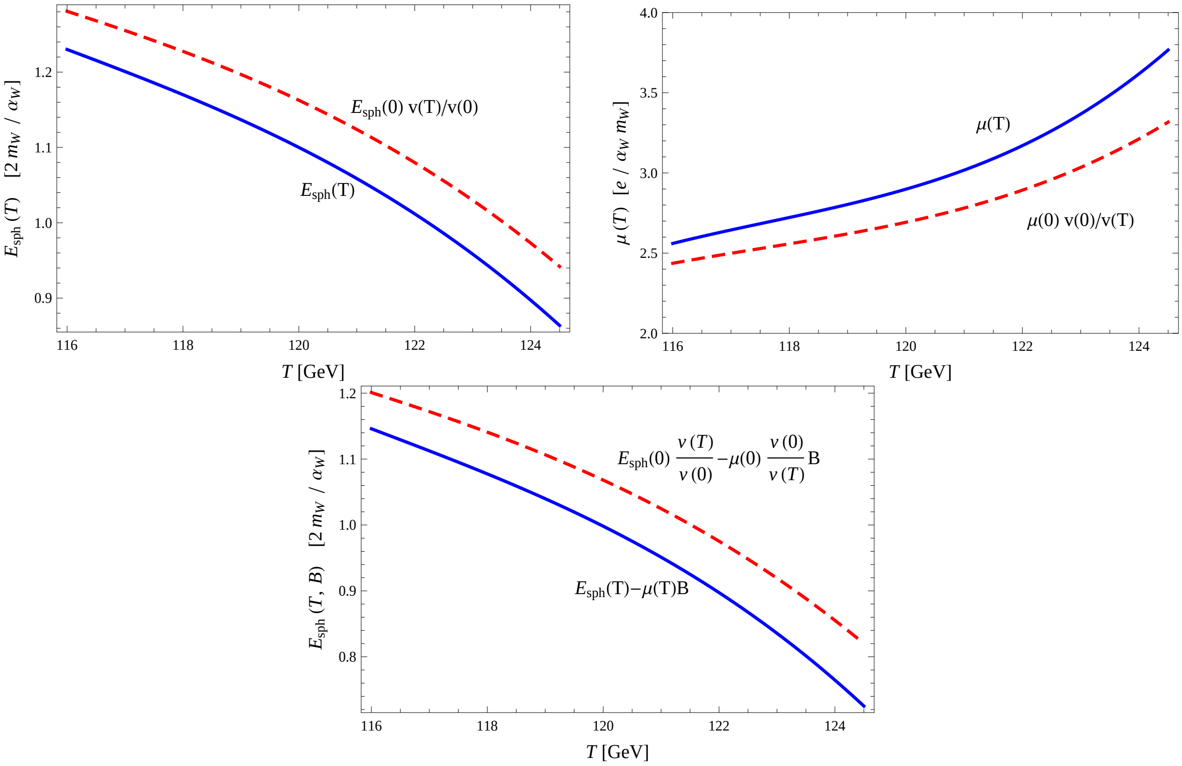

Figure 1: *The temperature dependence of the sphaleron energy (with and without magnetic field) and the magnetic dipole moment.* Top left panel: *sphaleron energy without magnetic field;* Top right panel: *sphaleron magnetic dipole moment;* Bottom panel: *sphaleron energy for* $B = 0.1\,T^2$*. We have fixed* $m_H = 120.2\,\text{GeV}$ *and* $\widetilde{m} = 10^3\,\text{TeV}$*. We show both the results of numerical calculations using the Higgs potentials at nonzero temperatures (blue solid line) and the result of a simple scaling relation (red dashed line).*

Beyond leading order in θ_w a non-linear dependence of the sphaleron energy on the magnetic field also arises. However for the range of magnetic fields that are relevant for our analysis the corrections to the linear approximation (3.2) are less than 5%, as discussed in Ref. [13].

To capture most of the temperature dependence of the sphaleron energy there exists a very common approximation which avoids resorting to solve for the sphaleron functions at $T \neq 0$. It consists in assuming that the whole temperature dependence is encoded in the expectation value $v(T)$. For vanishing magnetic field the scaling law is

$$E_\text{sph}^\text{scaling}(T) = E_\text{sph}(0)\frac{v(T)}{v(0)}\,, \tag{3.3}$$

which overestimates the correct energy by about 10%, as shown in Ref. [53] (top left panel of Fig. 1). On the other hand the dipole moment in Eq. (B.5) scales with the inverse of $v(T)$ and this is accurate to better than 15% (top right panel of Fig. 1). The scaling law for the total energy in presence of a magnetic field, Eq. (3.2), is given by the combination of the two scalings (bottom panel of Fig. 1). In this paper we have not made use of the scaling law but

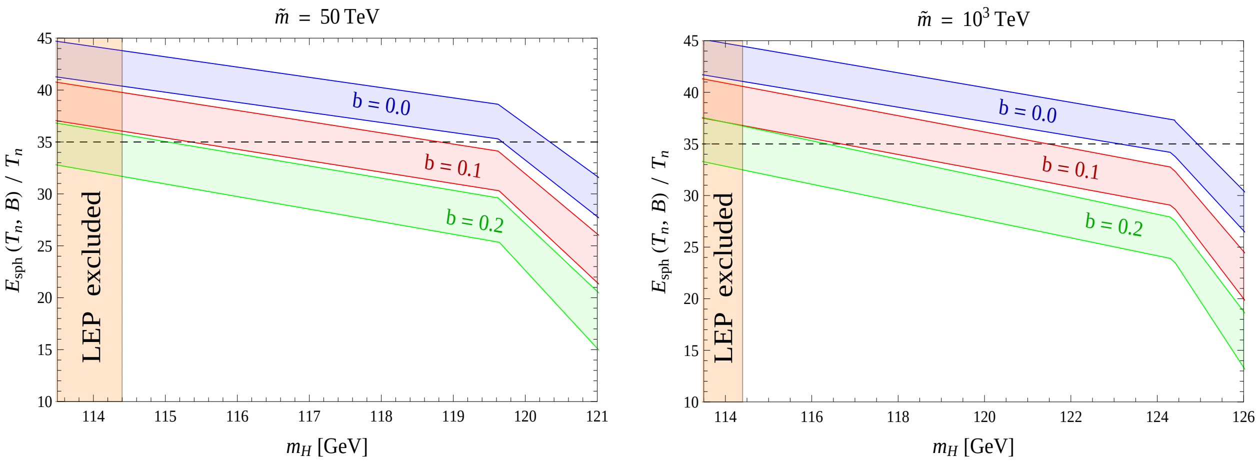

Figure 2: *The maximal sphaleron energy $E(T_n, B)/T_n$ achieved for a Higgs mass m_H evaluated with $\widetilde{m} = 50$ TeV* (left panel) *and $\widetilde{m} = 10^3\,\text{TeV}$* (right panel). *The horizontal dashed line corresponds to the requirement $E(T_n)/T_n \gtrsim 35$. The bands correspond to the uncertainty on the location of T_n in the interval $[T_c - 3.5\,\text{GeV}, T_c - 2\,\text{GeV}]$. The upper lines correspond to $T_n = T_c - 3.5\,\text{GeV}$. Different bands correspond to different values of $b = B/T_n^2 = 0.0, 0.1, 0.2$.*

instead directly computed the sphaleron solutions and the corresponding energy at non-zero temperature.

The condition for sphaleron transitions going out of equilibrium and not washing out the baryon asymmetry is [54] [3]

$$\frac{E_{\text{sph}}(T_n, B)}{T_n} \gtrsim 35\,, \tag{3.4}$$

where for simplicity we assume the magnetic field to become constant and sizable just after the bubble nucleations [4]. Implicitly, Eq. (3.4) provides a constraint on $V(H, \widetilde{t}_R, T_n)$ and for $B = 0$ it is roughly satisfied by the upper bounds $m_H \lesssim 127$ GeV and $m_{\widetilde{t}_R} \lesssim 120$ GeV [44]. Instead, for $B \neq 0$, these upper bounds become more stringent because of the negative contribution $E^{(1)}_{\text{dipole}}$ in Eq. (3.2). The numerical analysis will be done in the next section.

[3] One could prefer carrying out the analysis using a condition on $E_{\text{sph}}(T)/T$ at $T = T_c$ rather than $T = T_n$. We find that $E_{\text{sph}}(T_c)$ is lower than $E_{\text{sph}}(T_n)$ by about 15%, and the analogue of the bound (3.4) becomes $E_{\text{sph}}(T_c)/T_c \gtrsim 29$. If one adopted the scaling approximation (3.3), the $\mathcal{O}(10\%)$ overestimate of the sphaleron energy would partially cancel the mismatch between the energies computed at T_c and at T_n.

[4] Considering the B field to arise at a temperature T_B as low as T_f would tend to relax the bounds we will provide. However it seems realistic to consider that the main collisions occur at $T \simeq T_n$ and therefore we believe our final conclusion should be conservative. However lacking a detailed calculation on the generation of the magnetic field in the LSS first order phase transition we have not included the uncertainty in the determination of T_B in the plots.

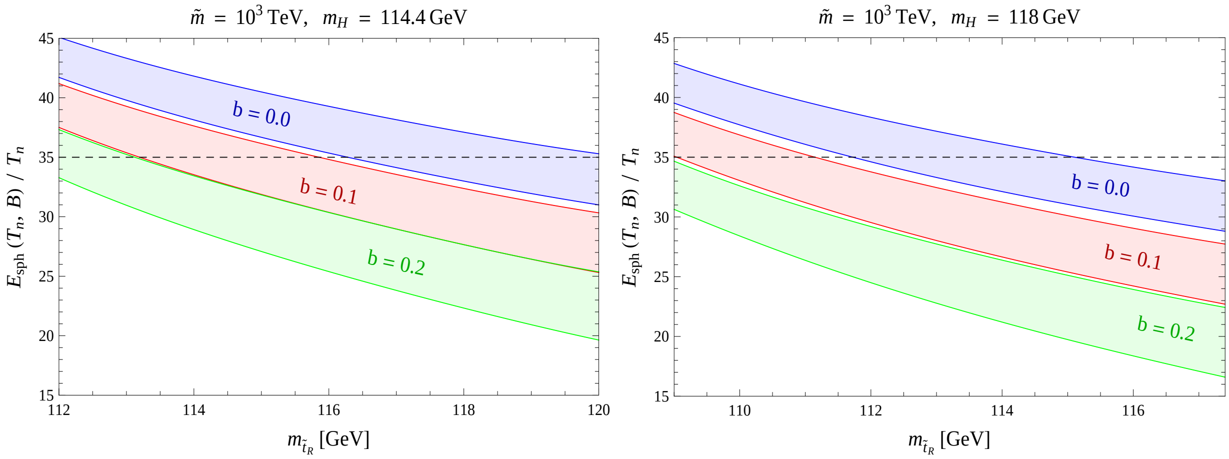

Figure 3: *The sphaleron energy $E(T_n, B)/T_n$ as a function of the maximal right-handed stop mass $m_{\tilde{t}_R}$ allowed for $\widetilde{m} = 10^3$ TeV and $m_H = 114.4$ GeV (left panel) or $m_H = 118$ GeV (right panel). The horizontal dashed line corresponds to the requirement $E(T_n)/T_n \gtrsim 35$. The bands correspond to the uncertainty on the location of T_n in the interval $[T_c - 3.5\,\text{GeV}, T_c - 2\,\text{GeV}]$. The upper lines correspond to $T_n = T_c - 3.5$ GeV. Different bands correspond to different values of $b = B/T_n^2 = 0.0, 0.1, 0.2$.*

4 Numerical results

We have considered a sample of points of the parameter space where we calculate $V(H, \widetilde{t}_R, T)$ of Eq. (2.1) and we use it to determine T_n by the bounce method [47, 48]. We observe that in the subset of points fulfilling the condition (3.4) with $B/T_n^2 \leq 0.2$ we get $T_c - 3.5\,\text{GeV} < T_n < T_c - 2\,\text{GeV}$ which we will translate into an error in the determination of T_n. Then we perform a wide scan in the LSS parameter space and at each point we calculate the effective potential of Eq. (2.1) and the correspondingly quantities $m_H, m_{\tilde{t}_R}, T_c$. From T_c we determine T_n with the error $[T_c - 3.5\,\text{GeV}, T_c - 2\,\text{GeV}]$ and subsequently $E_{\rm sph}(T_n)$.

Using this procedure we fix $\widetilde{m}$ and look for the maximal Higgs mass achieving a fixed value of $E_{\rm sph}(T_n, B)/T_n$. The result is shown in Fig. 2 for $\widetilde{m} = 50$ TeV and 10^3 TeV, and $b = B/T_n^2 = 0.0, 0.1, 0.2$. The value of the stop mass is conveniently chosen to maximize the Higgs mass. The slope of the curves changes, giving rise to the kinks observed in the figure, when the stop mass drops below the experimental bound $m_{\tilde{t}_R} \geq 95$ GeV [49] and therefore one needs to set the stop mass to its lower limit. The bands correspond to the uncertainty on the determination of T_n. Below the horizontal dashed line, Eq. (3.4) is not satisfied and EWBG in the LSS does not produce enough baryon asymmetry. We similarly repeat the scan by fixing m_H and looking for the maximal allowed stop mass consistent with a given value of $E_{\rm sph}(T_n)/T_n$. The outcome is presented in Fig. 3 for $\tilde{m} = 10^3$ TeV and $m_H = 114.4$ GeV (left panel) and $m_H = 118.0$ GeV (right panel). The maximal stop mass is now attained for the minimal Higgs mass and it gets lower when a magnetic field is present, although remaining above the experimental limit.

The previous results can be translated into absolute upper bounds on the produced mag-

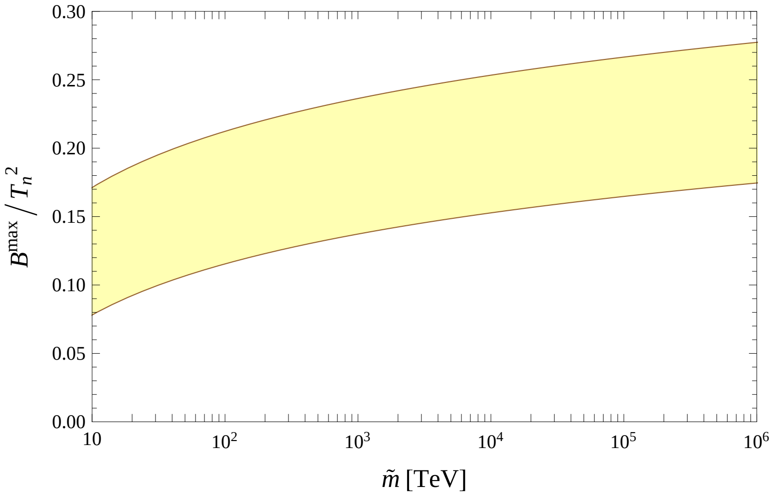

Figure 4: *The values of the magnetic field for which the maximal m_H is at the experimental bound. The band corresponds to the uncertainty on the location of T_n in the interval $[T_c - 3.5\,\mathrm{GeV}, T_c - 2\,\mathrm{GeV}]$. The upper line corresponds to $T_n = T_c - 3.5\,\mathrm{GeV}$.*

netic fields which are consistent with the requirement of EWBG in the MSSM. In Fig. 4 we show the values of the constant magnetic field generated at $T = T_n$ which pushes the maximum value of the Higgs mass required by successful EWBG in the LSS down to the present experimental bound. Again the band in Fig. 4 corresponds to the uncertainty on T_n. We see that even moderate values of magnetic fields generated during the EW phase transition could close the EWBG window.

5 Conclusions

In this paper we have pointed out that the sphaleron magnetic dipole moment couples to the magnetic field generated during a first order EWPT. This has the effect of lowering the sphaleron energy in the broken phase and consequently the baryon asymmetry is washed out more easily. We have not attempted to compute precisely the magnetic field generated at the phase transition, but rather we have considered it as a free parameter within a plausible range. In particular we have focused on the MSSM in the most favourable situation for EWBG, the light stop scenario, and explored the dependence of the sphaleron energy on the parameters of the model and on the magnetic field. We have shown that it is possible to have the baryon asymmetry preserved even in presence of a magnetic field by lowering the Higgs mass and/or the stop mass, or increasing the soft scalar mass $\widetilde{m}$. However even for moderate values of the magnetic field the required Higgs mass can fall below the present experimental bound and the window for EWBG in the MSSM gets closed.

The main conclusions of this paper are twofold. On the one hand our calculation cries out for an accurate determination of B and its evolution during the phase transition in order

to confirm if the magnetic field produced by the electroweak phase transition endangers the EWBG in the LSS, as our analysis seems to indicate. On the other hand, the magnetic field produced by phase transition bubbles threatens any model where the BAU is generated by a first order phase transition. In principle, any model of EWBG where the phase transition (evaluated at zero magnetic field) is never extremely strong should be jeopardized by the presence of a magnetic field.

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A Sphaleron solutions

Let us consider the classical finite energy configurations of the bosonic fields of the electroweak sector of the SM, in a gauge where the time components of the gauge fields are set to zero [12, 55]. The classical energy functional over configuration space, at temperature T, is

$$E_{\rm sph}(T) = \int d^3x \left[\frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}f_{ij}f_{ij} + (D_iH)^\dagger(D_iH) + V(H,T)\right] , \tag{A.1}$$

where

$$\begin{aligned} F^a_{ij} &= \partial_i W^a_j - \partial_j W^a_i + g\epsilon^{abc}W^b_i W^c_j , & \text{(A.2)}\\ f_{ij} &= \partial_i a_j - \partial_j a_i , & \text{(A.3)}\\ D_i H &= \partial_i H - \frac{1}{2}ig\sigma^a W^a_i H - \frac{1}{2}ig' a_i H , & \text{(A.4)} \end{aligned}$$

W^a_μ $(a = 1, 2, 3)$ and a_μ are the $SU(2)$ and $U(1)$ gauge fields, respectively, and H is the Higgs doublet. The gauge couplings of $SU(2)_L$ and $U(1)_Y$ are g and g', respectively, and the weak mixing angle is defined by $\tan\theta_{\rm w} = g'/g$. The potential V is defined as $V(H,T) \equiv V(H, \widetilde{t}_R = 0, T)$ as expressed in Eq. (2.1) and in Refs. [40, 44].

A.1 Case $\theta_{\rm w} = 0$

In this limit the $U(1)_Y$ gauge field a_i decouples and it may be set to zero. The sphaleron is a spherically symmetric configuration of gauge and Higgs fields. Let us consider the ansatz for

the fields [12, 52]

$$\begin{aligned} gW_i^a\sigma^a dx^i &= (1-f(\xi))F_a\sigma^a\,, & \text{(A.5)}\\ H &= \frac{v(T)}{\sqrt{2}}\begin{pmatrix}0\\ h(\xi)\end{pmatrix}\,, & \text{(A.6)}\end{aligned}$$

in terms of two radial functions $f(\xi)$ and $h(\xi)$ where the dimensionless distance $\xi \equiv gvr$ has been introduced, $\sigma^a(a=1,2,3)$ are the Pauli matrices and F_a are the 1-forms [52]

$$\begin{aligned} F_1 &= -2\sin\phi d\theta - \sin 2\theta\cos\phi d\phi\,, & \text{(A.7)}\\ F_2 &= -2\cos\phi d\theta + \sin 2\theta\sin\phi d\phi\,, & \text{(A.8)}\\ F_3 &= 2\sin^2\theta d\phi\,. & \text{(A.9)}\end{aligned}$$

After the redefinition

$$\widetilde{V}(h,T) \equiv V(H,T)|_{H\to(0,v(T)h/\sqrt{2})^T}\quad, \tag{A.10}$$

the sphaleron energy is a function of the radial functions

$$E_{\rm sph}^{(\theta_{\rm w}=0)}(T,B=0) = \frac{4\pi v(T)}{g}\int_0^\infty d\xi\left[4f'^2 + \frac{8}{\xi^2}f^2(1-f)^2 + \frac{1}{2}\xi^2 h'^2 + h^2(1-f)^2 + \xi^2\frac{\widetilde{V}(h,T)}{g^2v^4(T)}\right] \tag{A.11}$$

which is minimized by the solution of the variational field equations (prime denotes derivative with respect to ξ)

$$f'' - \frac{2}{\xi^2}f(1-f)(1-2f) + \frac{1}{4}h^2(1-f) = 0\,, \tag{A.12}$$

$$h'' + \frac{2}{\xi}h' - \frac{2}{\xi^2}h(1-f)^2 - \frac{1}{g^2v(T)}\frac{\partial\widetilde{V}(h,T)}{\partial h} = 0\,. \tag{A.13}$$

with boundary conditions $f(0)=h(0)=0,\ f(\infty)=h(\infty)=1$.

A.2 Case $\theta_{\rm w}\neq 0$

When $\theta_{\rm w}\neq 0$ the sphaleron is not spherically symmetric but only axisymmetric. The most general ansatz requires seven independent functions of the spherical coordinates r and θ [56]. However since the dependence on θ is very mild an excellent approximation to the exact solution is provided by an ansatz in terms of only four scalar functions of r [52]

$$\begin{aligned} g'a_i\,dx^i &= (1-f_0(\xi))F_3\,, & \text{(A.14)}\\ gW_i^a\sigma^a dx^i &= (1-f(\xi))\left(F_1\sigma^1 + F_2\sigma^2\right) + (1-f_3(\xi))F_3\sigma^3\,, & \text{(A.15)}\\ H &= \frac{v(T)}{\sqrt{2}}\begin{pmatrix}0\\ h(\xi)\end{pmatrix}\,. & \text{(A.16)}\end{aligned}$$

Now the sphaleron energy reads

$$
\begin{aligned}
E_{\rm sph}(T) \;=\;& \frac{4\pi v(T)}{g}\int_0^\infty d\xi\left\{\frac{8}{3}f'^2+\frac{4}{3}f_3'^2+\frac{8}{\xi^2}\left[\frac{2}{3}f_3^2(1-f)^2+\frac{1}{3}\left(f(1-f)+f-f_3\right)^2\right]\right.\\
&+\frac{4}{3}\left(\frac{g}{g'}\right)^2\left[f_0'^2+\frac{2}{\xi^2}(1-f_0)^2\right]+\frac{1}{2}\xi^2 h'^2+h^2\left[\frac{1}{3}(f_0-f_3)^2+\frac{2}{3}(1-f)^2\right]\\
&\left.+\xi^2\frac{\widetilde{V}(h,T)}{g^2v(T)^4}\right\}\,, \qquad\qquad (A.17)
\end{aligned}
$$

which is minimized by the solutions of the variational equations

$$
f''+\frac{2}{\xi^2}(1-f)\left[f(f-2)+f_3(1+f_3)\right]+\frac{1}{4}h^2(1-f)=0\,, \qquad (A.18)
$$

$$
f_3''-\frac{2}{\xi^2}\left[3f_3+f(f-2)(1+2f_3)\right]+\frac{1}{4}h^2(f_0-f_3)=0\,, \qquad (A.19)
$$

$$
f_0''+\frac{2}{\xi^2}(1-f_0)-\frac{g'^2}{4g^2}h^2(f_0-f_3)=0\,, \qquad (A.20)
$$

$$
h''+\frac{2}{\xi}h'-\frac{2}{3\xi^2}h\left[2(1-f)^2+(f_0-f_3)^2\right]-\frac{1}{g^2v(T)^4}\frac{\partial\widetilde{V}(h,T)}{\partial h}=0\,. \qquad (A.21)
$$

with boundary conditions $f(0)=f_3(0)=h(0)=0$, $f_0(0)=1$ and $f(\infty)=f_3(\infty)=f_0(\infty)=h(\infty)=1$. For $\theta_{\rm w}\to 0$ one recovers the solutions of the previous subsection: $f_0(\xi)\to 1, f_3(\xi)\to f(\xi)$.

B Sphaleron magnetic dipole moment

A magnetic dipole moment for the sphaleron arises when $\theta_{\rm w}\neq 0$ [12]. The $U(1)_Y$ gauge field a_i is not decoupled, as in the case $\theta_{\rm w}=0$, and it cannot be set to zero because it is sourced by the current

$$
J_i=-\frac{1}{2}ig'\left[H^\dagger D_iH-(D_iH)^\dagger H\right]\,, \qquad (B.1)
$$

through the field equation $\partial_j f_{ij}=J_i$. The shift in the sphaleron energy due to the dipole moment interaction is given by [12]

$$
E_{\rm dipole}=-\int d^3x\; a_i\,J_i\,, \qquad (B.2)
$$

which is negative. At the first order in $\theta_{\rm w}$ one can neglect a_i in the current which then becomes

$$
J_i^{(1)}=-\frac{1}{2}g'v^2\frac{h^2(gvr)(1-f(gvr))}{r^2}\epsilon_{3ij}x_j\,, \qquad (B.3)
$$

where the sphaleron radial functions $f(\xi),h(\xi)$ ($\xi=gvr$) are the solutions of Eqs. (A.12)-(A.13) at $T\neq 0$. Using the vector potential of a constant magnetic field B along the $\hat{z}$-axis, $a_i=-(B/2)\epsilon_{3ij}x_j$, the dipole energy reads

$$
E^{(1)}_{\rm dipole}(T,B)=-\mu(T)\,B\,, \qquad (B.4)
$$

where

$$\mu(T) = \frac{2\pi}{3}\frac{g'}{g^3 v(T)}\int_0^\infty d\xi \xi^2 h^2(\xi)[1-f(\xi)]\,, \tag{B.5}$$

and the temperature dependence also resides in the radial functions $f(\xi), h(\xi)$.

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